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A COORDINATION POLICY FOR THE NATO SEASPARROW MISSILE AND THE ROLLING AIRFRAME MISSILE USING DYNAMIC PROGRAMMING

by

Arthur Paul Drennan III

September, 1994

Thesis Advisor: Alan R. Washburn

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by

Arthur Paul Drennan III
Lieutenant, United States Navy
B.S., United States Naval Academy, 1988

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Author: ______________________
Arthur Paul Drennan III

Approved by: ______________________
Alan R. Washburn, Thesis Advisor

____
Robert F. Dell, Second Reader

____
Peter Purdue, Chairman
Department of Operations Research

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The reader is cautioned that computer programs developed in this thesis may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

Anti-air warfare (AAW) has long been a top priority for the United States Navy, in terms of developing tactical proficiency and in acquiring the right systems to defend ships from attack. Several recent trends have focused attention on the need for improved self-defense capability for many fleet units. Of these, the most fundamental are the changing foci of Navy operations as outlined in ...From the Sea, the Navy and Marine Corps White Paper [Ref 1: pp. 3-5], and the proliferation of Anti-Ship Missiles (ASMs) worldwide.

Littoral warfare as defined in ... From the Sea [Ref 1: p. 5] contains several aspects that force the development of improved ASM self-defense capabilities. By moving naval operations in proximity to land, the number of potential aggressors capable of launching an ASM attack has increased, while the reaction time available has decreased. Several modern ASMs are capable of being fired from mobile land-based launchers, and many nations who lacked the ability to threaten a battle group in the open ocean are capable of launching a coordinated strike at ships operating near their coasts.

This potential for increased exposure is coupled with the proliferation of modern ASMs worldwide. Table 1.1 illustrates the countries known to have acquired the Exocet and Otomat missiles, only two of many capable ASMs that must be considered.[Ref 2: p. 87]. This proliferation issue is not limited simply to increased numbers of ASMs, but to different kinds of threat missiles as well. Intelligence estimates envision three times as many ASM types by the turn of the century as existed in the early 1970's when cruise missile ASM development began in earnest. [Ref 3: p. 22]

These developments have not occurred overnight, nor is the Navy without a self-defense capability. The Phalanx Close-In Weapons System (CIWS) and NATO Sea Sparrow Missile Systems (SEASPARROW) have been deployed for active self-defense for some time. Current efforts at improving self-defense for non-AAW ships have focused on improvements to the above systems and the introduction of the Rolling Airframe Missile (RAM). RAM has the advantage of using passive guidance for homing, which

obviates the need for expensive fire control systems. This is an attractive feature when upgrading amphibious vessels or combat logistics force ships.

Recent efforts have focused on fusing these three systems into an integrated Ship Self-Defense System (SSDS). This provides a layered self-defense capability, as well as flexible response since not all threat missiles can be engaged by any one system. The task of integrating these different systems is not trivial. Each system is fundamentally different in its method of defense, and each has unique features that drive its employment.

MISSILE	COUNTRY
EXOCET	Argentina, Bahrain, Brazil, Brunei, Cameroon, Chile Colombia, Ecuador, Egypt, France, Germany, Greece, Indonesia, Iran, Iraq, Kuwait, Libya, Malaysia, Morocco, Nigeria, Oman, Pakistan, Peru, Philippines, Qatar, Singapore, South Korea, Thailand, Tunisia, UK
OTOMAT	Egypt, France, Iraq, Italy, Kenya, Libya, Nigeria, Peru, Saudi Arabia, Venezuela

TABLE 1.1

This thesis seeks to develop a policy for coordination among the SEASPARROW, RAM and CIWS systems as they are intended for installation on the DD-963 Spruance class destroyer. The analysis is carried out from the perspective of the defender operating without mutual support from ships in company. The objective of the defender is to maximize the probability of survival for a given ASM threat scenario using the three weapon systems in a coordinated manner.

Chapter II describes these weapon systems, highlighting the unique features of each that drive the utilization of these weapons. Previous modeling approaches for this installation are described for purposes of comparison. The threat scenario and an overview of the coordination problem are developed in Chapter III. This chapter also details the assumptions made in modeling this system.

Chapter IV presents the formulation of the Dynamic Program used to model this system. Chapter V reports the test plan for analyzing the coordination policy and the results of the model. Results for the test plan demonstrate the range of parameter values for which these results hold.

Chapter VI draws conclusions from these results, makes recommendations for application of these results and documents potential follow-on research and excursions.

II. BACKGROUND

This chapter details the characteristics of the weapons systems intended for the DD-963 Spruance destroyer and discusses previous modeling efforts that assumed uncoordinated assignment of weapons. The DD-963 Ship Self Defense System (SSDS) configuration includes the NATO SEASPARROW Missile System (SEASPARROW), the Rolling Airframe Missile (RAM) system and the Phalanx Close-In Weapon System (CIWS). Each of these systems has unique characteristics that drive their tactical employment.

A. SEASPARROW MISSILE SYSTEM CHARACTERISTICS

The SEASPARROW evolved from the air-to-air AIM-7 SPARROW missile as a point defense weapon for surface ships in the early 1960's. Initially only used by the United States, it was updated and installed in 1973 in the navies of 11 other allied countries. Table 2.1 details the physical features of the weapon. [Ref 2: p. 254]

NATO Sea Sparrow (NSSMS)			
Dimensions:		**Performance:**	
Length	3.66m	Range, Max	7.86mi
Diameter	20.3cm	Range, Min	0.5mi
Span	1.02m	Altitude	15,000m
Weight	228kg	Speed	1.3mach
Guidance: Inverse monopulse semi-active radar seeker			

TABLE 2.1

The key feature of the SEASPARROW system is the requirement for continuous illumination of the selected threat throughout the flight of the missile. Since the

SEASPARROW is a semi-active system, failure to properly illuminate the incoming missile will result in failure of the SEASPARROW to intercept the target. The DD-963 installation of the SEASPARROW provides one illumination radar, and one eight missile launcher. Consequently the SEASPARROW is a one-threat-at-a-time system. This feature is the critical factor in employing the system. On the other hand, SEASPARROW is not reliant on a cooperatively radiating target, as is the Rolling Airframe Missile.

B. ROLLING AIRFRAME MISSILE CHARACTERISTICS

The Rolling Airframe Missile (RAM) was developed jointly with the Federal Republic of Germany. The system satisfies the need for a quick reaction, high firepower self defense missile system complementary to existing systems that can be installed on a variety of U.S. Navy ship classes. Table 2.2 details the physical characteristics of the RAM system [Ref 2: p. 204].

Rolling Airframe Missile (RAM)			
Dimensions:		**Performance:**	
Length	2.79m	Range, Max	5mi
Diameter	12.7cm	Range, Min	0.5mi
Span	43.4cm	Altitude	12,000m
Weight	73.6kg	Speed	2.0mach
Guidance: Dual mode passive RF midcourse, IR terminal			

TABLE 2.2

The critical feature of this weapon that limits its employment tactically is the use of passive guidance. The system uses a dual mode guidance system, homing initially on passively received emissions from the seeker of the inbound threat missile. During the

intercept flight, the missile will switch guidance modes to infrared once the heat signature of the threat missile exceeds some threshold value.

The advantage of passive guidance is that ships do not require sophisticated tracking and guidance radars to employ the RAM system. Passive guidance of RAM is the driving factor in its employment. Tactically, passive guidance means that the missile cannot be designated against a specific target with confidence, and RAM requires a radiating target. Additionally, the use of infrared seekers for terminal homing creates the need to limit the speed at which the launcher fires rounds at incoming targets. This prevents the missile from acquiring and homing on the previous missile's rocket thrust. Note that while the range of the RAM is shorter than for SEASPARROW, the missile is significantly faster.

The DD-963 installation of the Rolling Airframe Missile includes one 21 round trainable launcher.

C. CLOSE-IN WEAPON SYSTEM CHARACTERISTICS

The Phalanx Close-In Weapon System (CIWS) was developed as a stand alone point defense system in the 1970's and deployed on active units in 1980. The purpose of the Phalanx is to provide last ditch defense against anti-ship missiles. Table 2.3 provides the physical characteristics of the CIWS [Ref 4].

In its primary mode, CIWS provides continuous surveillance and defense within its engagement envelope, independent of other ship systems. CIWS is an automatic gun system which combines its own surveillance radar, fire control system and 20mm Gatling gun to act as a stand-alone defense installation. The CIWS uses closed loop spotting to achieve multiple hits on a target missile. Closed loop spotting works by tracking the target and the bullet stream and reducing the separation between the two until the projectiles are hitting the target.[Ref 4]

The DD-963 installation includes two MK15 CIWS mounts, located on the forward superstructure, starboard side and on the aft superstructure, port side. This provides 360 degree coverage from the two mounts.

Close-In Weapon System (CIWS)			
Specification:		**Performance:**	
Calibre	20 mm	Range, Max	1mi
Muzzle Velocity	1030 m/s	Range, Min	0.0mi
		Rate of fire	3000 rds/min
Weight	5240kg		
		Magazine	1000 rds
Guidance:			
Closed loop spotting, pulse doppler radar			

TABLE 2.3

D. SYSTEM MODELS

The installation of these three weapons on the DD-963 has been modeled before. This study was prompted by a request from Hughes Missile Systems Company (HMSC) to investigate coordination policies for these weapons. HMSC conducted a study of the potential benefit of coordinating RAM and CIWS. They concluded that since CIWS effectiveness in the overlapping engagement zone was negligible compared to RAM, there was no advantage to imposing a coordination policy on these two weapons. [Ref 5]

Hughes has also developed a Monte Carlo simulation that models the DD-963 weapons configuration among others. The simulation is written in C++ and runs on a PC. It gives the option of an animated graphic display where the engagement can be observed as it occurs. This simulation uses an uncoordinated policy for determining weapon assignments. [Ref 6]

The Hughes model reveals some drawbacks to assigning weapons independently. Of particular interest are engagements where the SEASPARROW system and the RAM system engage the same target simultaneously. This can frequently result in the RAM system defeating the threat missile, while the SEASPARROW salvo is still in flight. Consequently, the SEASPARROW has been allocated to a threat which was defeated by RAM and thus has missed an opportunity to engage some other target. By applying an uncoordinated firing doctrine, the combined system does not seek to deconflict targets for the component weapons and thus fails to ensure resources are allocated efficiently.

A dynamic programming approach to modeling the system was developed by LT Roger Powell for his Master's Thesis [Ref 7]. He also modeled the component weapons with an independent assignment doctrine. His analysis concluded that the optimal policy for the combined SEASPARROW and RAM defense was to maximize the number of engagement opportunities to achieve the highest probability of survival. In addition, his analysis showed that allocating fewer resources to the initial salvos improved survivability, as does increasing the volley of fire as the threat closes.

Both of these approaches reveal several drawbacks to allocating RAM and SEASPARROW assets independently. By far the most debilitating result is that the uncoordinated assignment fails to ensure that all targets are engaged. The uncoordinated policy allows the SEASPARROW to engage a threat missile, then direct the RAM to fire on the same threat missile. This doctrine runs counter to Powell's conclusion that maximizing opportunities is the optimal goal of weapon assignment policy. Intuitively, assigning two different weapons to the same target at the same time is not the best decision, particularly in a self defense scenario. The timelines at these close ranges are compressed, and leave little room for faulty decision making or wasted assets. The task is to determine a policy that promotes mutual support among RAM and SEASPARROW assignments.

III. PROBLEM DESCRIPTION

Previous studies that model the DD-963 self defense installation fail to coordinate the assignment of RAM and SEASPARROW resources for mutual support. The goal of this study is to develop a coordination scheme for these two systems, and test the results to determine the impact of this policy on the assignment decisions of the combined system. The principle measure of effectiveness is the probability of survival against a given threat scenario.

The following structure is used to develop an abstract description of the problem, and the approach to modeling it. First, the DD-963 system is described, focusing on the possible interactions of the component weapons and how they affect the problem. Second, the baseline threat scenario of interest is developed. Finally, the assumptions made in order to facilitate modeling the system are presented and discussed.

A. SYSTEM DESCRIPTION

The physical characteristics of the component weapons have been described. Each weapon is unique, with different capabilities and limitations. The weapons have been installed with an eye toward layered defense and flexible response. The area of interest for this study is the range in which both RAM and SEASPARROW are capable of engaging an inbound threat. The Hughes Missile Systems Company study on RAM and CIWS concluded there was no benefit to coordinating between these two component systems. Consequently, CIWS is not a candidate for coordination, and serves as the "garbage collector" of the combined system.

Figure 3.1 illustrates the engagement zones for the system. The diagram shows that there is a zone in which SEASPARROW can act independently, beyond RAM range. RAM as a passive system is not limited in the number of threats it can simultaneously engage. This is precisely the defining limitation of the SEASPARROW system, particularly in the DD-963 installation which includes only a single illuminator. Provided the threat missiles are not themselves passively guided, the only limitation on the number

RAM can engage is the limit on launch frequency imposed by RAMs infrared seeker. Additionally, the RAM launcher has a magazine of 21 missiles compared to only eight SEASPARROW. Thus, the defense will rely primarily on RAM, with the SEASPARROW as a supporting asset, and CIWS to handle any leakers.

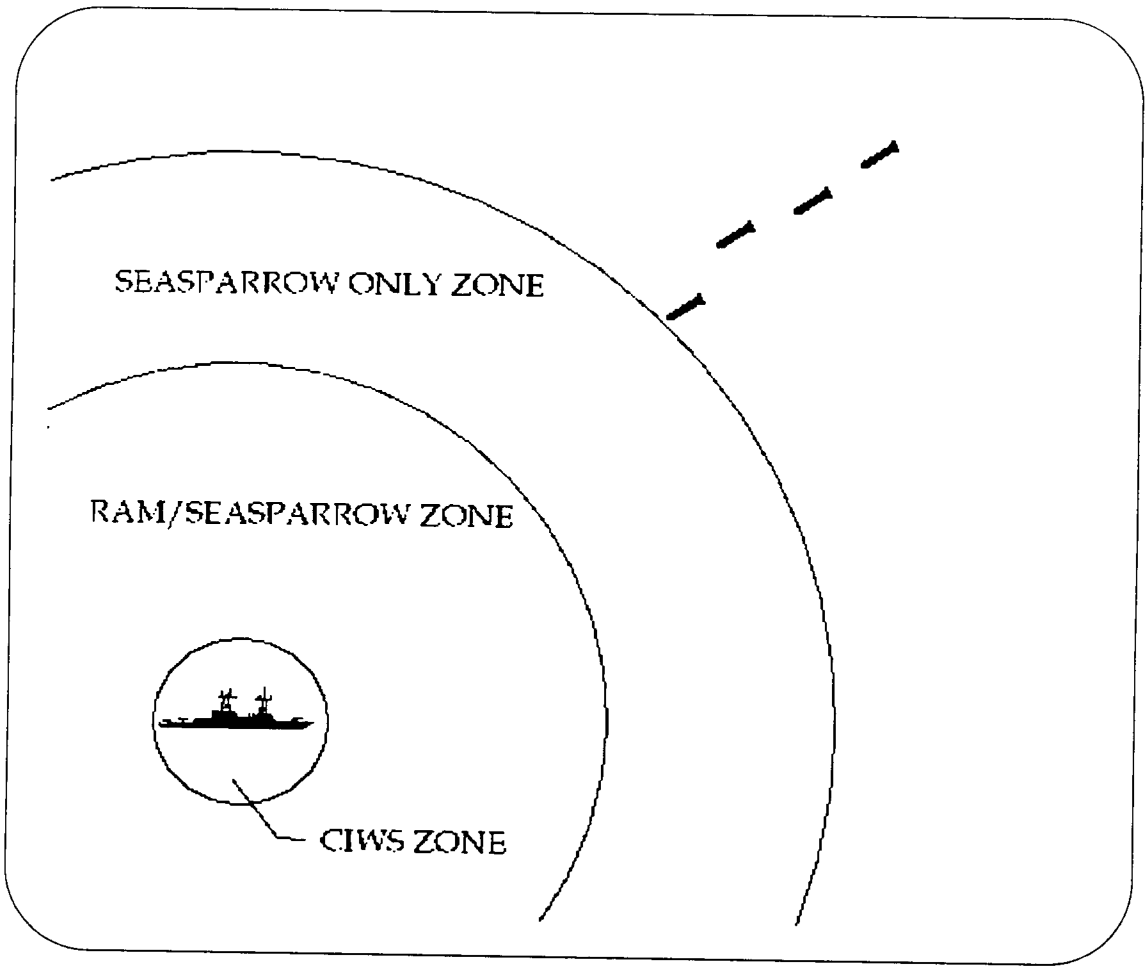

Figure 3.1

The assignment decision rests with SEASPARROW. RAM in general cannot be assigned to a particular target with reliability. RAM will always engage the lead target, CIWS has been shown to perform best when left as an independent asset, but SEASPARROW with semi-active guidance offers the flexibility of choosing which target in the stream it will attack. SEASPARROW is not limited to striking only the lead target,

and in this capability lies the potential for mutual support. Coordination policy then starts by determining which target the SEASPARROW should engage.

The contention here is that RAMs main limitation is on frequency of fire. Indeed were it not for the limitation imposed by its launcher delay, RAM would be capable of handling most active threat scenarios unilaterally. However, because of the fratricide delay the worst condition is for threat missiles to arrive so closely that RAM will be unable to cycle fast enough to engage all targets. In other words, the RAM launcher might become momentarily saturated by closely spaced threat missiles. SEASPARROW should consequently attack these tight groups of threat missiles in order to ease the burden on RAM. In order to develop this logic further, the specific threat scenario is outlined below.

B. THREAT SCENARIO

The vast majority of anti-ship cruise missiles are subsonic [Ref 2: pp. A2-A24]. Of these the representative threat missile is the French made Exocet. Apart from the large number of these weapons exported, the Exocet has the best features of a sea skimming subsonic cruise missile: low altitude, low radar cross section, and high reliability. It is also the threat Powell and Hughes modeled. Table 3.1 details the physical characteristics of the three versions of the Exocet missile [Ref 2: p. 66].

For this study passive anti-ship missiles, which include both infrared guided and anti-radiation homing missiles, are not contemplated because these are not eligible for RAM to defend against. These threats do not have a Radio Frequency (RF) emission for RAM to guide on, and thus are not suitable for a coordination decision. Additionally, diving cruise missiles are not considered due to their relatively limited proliferation globally.

The nominal threat scenario is a stream attack of four Exocet missiles arriving in a 20 second time window on one bearing. The single bearing scenario is simpler to model because it obviates the need to calculate launcher slew rates as the threats are all on the

same bearing. It is also most stressful to RAM, since the fratricide potential is maximized. Figure 3.1 illustrates a stream attack.

EXOCET ASCM							
Dimensions:	**MM38**	**MM40**	**AM39**	**Performance:**	**MM38**	**MM40**	**AM39**
Length	5.21m	5.78m	4.69m	Range, Max	26mi	43mi	30-43mi
Diameter	35.0cm	35.0cm	35.0cm	Range, Min		Unknown	
Span	1.004m	1.135m	1.004m	Altitude		Sea Skimming	
Weight	735kg	850kg	652kg	Speed		0.93mach	
Guidance: Inertial autopilot midcourse with X band monopulse active terminal homing							

TABLE 3.1

C. PROBLEM ASSUMPTIONS

Assumptions are divided into general system or scenario level assumptions, and those concerning each of the three component weapon systems: SEASPARROW, RAM and CIWS.

1. General System and Scenario Assumptions

All threat missiles in a raid are of the same type. Because the intent is to determine coordination policy, the combined system is not subject to bias errors from the detection sensors which could confound targeting. There are no potential shared targeting errors among missile salvos. Missiles in flight are assumed to behave independently of one another.

The defending DD-963 platform has already maneuvered to unmask batteries and no further maneuvers are required to engage the stream attack. All threat missiles are

targeted at the defending platform, so there are no crossing target geometries to consider. All attacks are conducted in an electronic warfare clear environment. These assumptions all serve to reinforce the position that the intent is to assess weapon assignment coordination decisions.

2. Assumptions Concerning SEASPARROW

All threat missiles are detected at problem start, and the character of the attacking missile raid is known prior to any possible SEASPARROW assignment decision. The SEASPARROW system is fully operational throughout the scenario, subject only to the following:

- the specified Probability of Kill (PK) at problem start,
- the specified magazine loadout at problem start, and
- the system has only a single illuminator.

SEASPARROW is subject to a minimum delay between engagements. This delay consists of time for track to launch delay, battle damage assessment time, illuminator tie-up time and other processing delays. For this analysis ten seconds is used. This is not the actual delay time for the system, but a representative figure. [Ref 8]

The single shot Probability of Kill (PKss) is constant throughout the engagement envelope. In other words, the SEASPARROW PK is not dependent on range. Additionally, the SEASPARROW has constant velocity throughout its engagement envelope. No attempt has been made to impose a salvo doctrine on SEASPARROW. The system has the option of firing a single round salvo, a dual round salvo or holding fire altogether.

3. Assumptions Concerning RAM

The central feature of the Rolling Airframe Missile is the required delay between missiles. RAM launcher cycle time includes the mechanical delay required to launch a successive round and a fratricide delay. The fratricide delay prevents the rocket thrust from a RAM from satisfying the Infrared terminal homing criteria of a following round. For purposes of the baseline scenario, the launcher delay is five seconds. It should be

noted that this is a representative value, and not indicative of RAMs true performance [Ref 8].

All threat missiles are assumed to satisfy RAM engagement criteria prior to closing within RAM range. Threat seekers are on and radiating at sufficient power to allow passive RF homing for RAM, and Infrared signatures are above RAMs threshold level to allow terminal guidance. RAM's PKss is considered constant throughout its engagement envelope and not a function of intercept range.

RAM firing policy is to fire single RAMs successively at the closest threat until it has been neutralized or has penetrated RAMs minimum range, before subsequent threats are engaged. RAMs that miss the nearest target do not acquire later targets in the stream. RAM is not allowed to fire at threats that are currently assigned to the SEASPARROW system.

From Tables 2.1 and 2.2, RAM is nearly twice as fast as SEASPARROW, Mach 2.0 versus Mach 1.3 respectively. RAM engagement outcomes are determined upon a RAM launch. This is the most contentious assumption made in order to model this system, but it is a necessary one. The intention of the model is to determine which SEASPARROW assignment doctrine maximizes survival. There are numerous events occurring while a SEASPARROW salvo is in flight, including the launch of RAMs and CIWS salvos. The complexity involved in trying to model all the potential events while each RAM is in flight multiplied by all possible SEASPARROW decisions quickly expands the scope of the problem beyond reason. Determining RAM engagement outcomes upon launch precludes the need to track RAMs still in flight when the SEASPARROW intercept occurs. Thus subsequent SEASPARROW decisions are made with the outcome of all RAM salvos known.

4. Assumptions Concerning CIWS

The CIWS model is the least developed. The goal is to study SEASPARROW policy, and CIWS is an independent system. As such its only purpose in the model is to

handle leakers which penetrate beyond the SEASPARROW and RAM minimum engagement range.

All threat missiles are detected by CIWS prior to entering maximum engagement range. CIWS is fully operational subject only to the PK specified at the start of the problem. The CIWS magazine is considered to be infinite to preclude the necessity of tracking remaining magazine capacity, and because the nominal scenario has only four threat missiles. The expected number of leakers is well below the CIWS magazine capacity.

CIWS cannot engage more than one target simultaneously, and must enjoy a certain spatial separation between inbound threats in order to engage sequentially. These aspects of CIWS are not modeled. The CIWS PK is modeled as 0.5. This is a representative value and not necessarily indicative of true performance.

IV. DYNAMIC PROGRAMMING FORMULATION

The decision tree for the optimal assignment of the SEASPARROW missile is complicated and expands rapidly. Figure 4.1 illustrates this complexity. For every possible time since the first threat missile enters SEASPARROW engagement range a decision regarding which target should be engaged, and with what salvo size must be made. The possibility of not firing SEASPARROW because of some potential advantage to waiting must also be included. As the diagram illustrates, for a stream of four threat missiles this yields nine potential SEASPARROW assignment decisions to investigate: a salvo of one for each target (four possibilities), a salvo of two for each target (four possibilities) or a salvo of zero (one possibility). The SEASPARROW can engage only one target at a time; otherwise there would be even more alternatives.

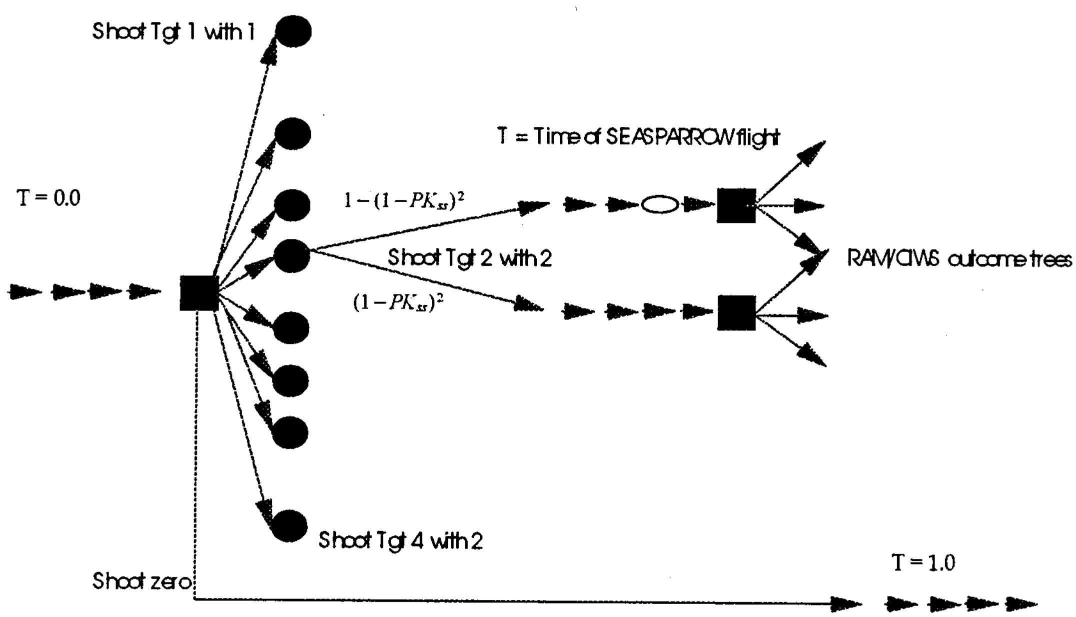

Figure 4.1

For each of these SEASPARROW decisions there are a multitude of other potential interactions to investigate. Threats can enter the RAM envelope during the time of flight of the SEASPARROW salvo and must be resolved as RAM engagements. Threats can travel all the way to the defending ship during the SEASPARROW salvo, and

must be resolved as RAM and CIWS engagements as well. For every SEASPARROW decision another decision tree branches off. The complexity of this tree is determined by the duration of the SEASPARROW salvo flight time, and the distance to the targets when the SEASPARROW assignment was made.

Dynamic Programming offers a method based on recursive calculations to handle the magnitude of this decision tree efficiently. The structure outlined in the rest of this section is used to develop the dynamic program that models this system: the SEASPARROW Coordinated Assignment Model (SCAM). SCAM optimally assigns SEASPARROW salvos to maximize the probability of survival of the defending ship. The decision points in the threat scenario where SEASPARROW decisions are made are graphically depicted and defined. These decision points lead to a convenient partition of the threat stream into three mutually exclusive categories of missiles for each SEASPARROW assignment decision. These categories are described. The state variables used in SCAM are defined, and the model parameters are stated and described. The recursive calculations for each threat missile category are detailed, and the SEASPARROW assignment recursion is developed in detail. Finally, some specifics of the implementation are briefly described.

The decision points for SEASPARROW occur whenever a threat missile is in range of the SEASPARROW system and the system is not currently engaging another target. That is, SEASPARROW as a single threat at a time weapon can only be allocated to a single target. Decisions about SEASPARROW assignments are made at the conclusion of a previous assignment, or if the system is idle when a threat enters the engagement envelope. Figure 4.2 depicts several assignment opportunities for SEASPARROW. The top image shows the threat stream has entered SEASPARROW range, but is outside RAM range. The decision under investigation is to fire the SEASPARROW at the third threat missile. Based on this decision and the known velocities of the SEASPARROW and the threat, the intercept range for this decision is calculated. The center image shows the threat stream disposition at intercept. Now the

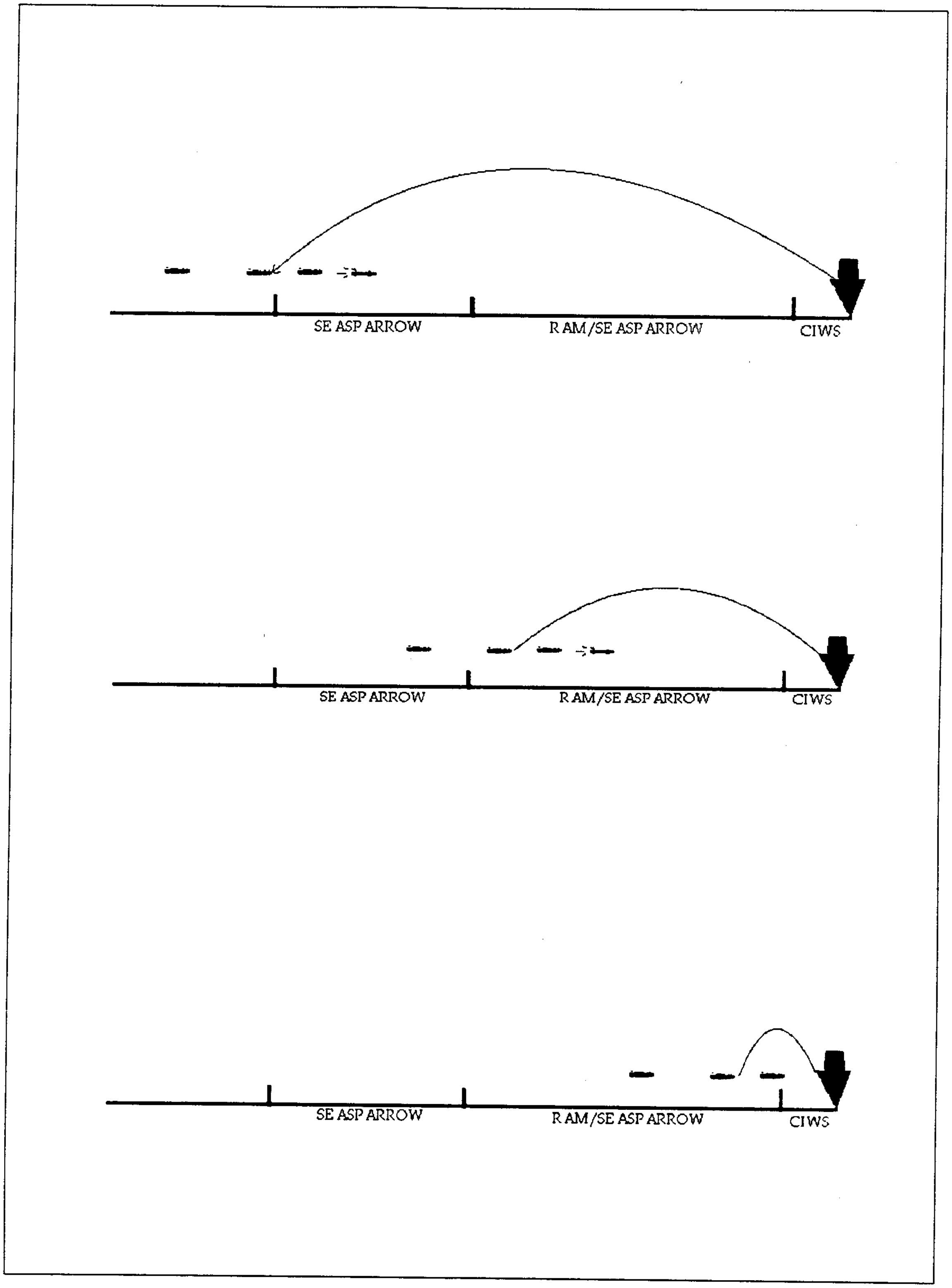
SE ASP ARROW
R AM/SE ASP ARROW
CIWS
SE ASP ARROW
R AM/SE ASP ARROW
CIWS
SE ASP ARROW
R AM/SE ASP ARROW
CIWS

Figure 4.2

first three missiles in the stream are shown to have entered RAM range during the SEASPARROW's flight to intercept target three. In this case the RAM has the opportunity to engage the lead two missiles. The first two missiles in the stream are classified as Eligible to RAM. The third threat missile is not as it is the target of the SEASPARROW system. The number of opportunities RAM has to neutralize the Eligible targets is determined by calculating the time of flight for the SEASPARROW to intercept target three, and dividing by the launcher cycle time for the RAM system. This yields the number of RAM salvos which can be directed against Eligible targets during the SEASPARROW intercept. In the center case at intercept the worst has happened and both SEASPARROW and RAM have missed all targets.

Now consider the middle case. Treat this as a new SEASPARROW assignment opportunity that requires evaluation of a SEASPARROW decision. Again the system is evaluating the case where SEASPARROW is assigned against the third missile in the stream. At the time of the assignment the lead three targets satisfy RAM engagement criteria. The bottom illustration depicts the threat stream condition at intercept for this assignment decision. In this case, the lead target had enough time to close all the way to the defending ship. The lead target is classified as a Critical target, one which must be defeated during the current time or the ship will suffer a hit. All remaining threats now satisfy RAM engagement criteria. Threats two and four are considered Eligible to RAM and threat three is the designated SEASPARROW target. Again the worst case has happened and SEASPARROW and RAM have failed to neutralize threats two, three and four. That the ship still survives indicates that RAM or CIWS successfully defended against the lead missile, the Critical target.

This diagram thus demonstrates the three exclusive categories of targets and how they are determined based on a SEASPARROW assignment decision. Given the same starting positions, a SEASPARROW assignment decision to engage the lead target would have resulted in a different classification of the threat missiles. The SEASPARROW assignment determines the classification of threats in the stream and the number of RAM

salvos that will be expended during the SEASPARROW salvo time of flight. It is clear from these diagrams that for any SEASPARROW assignment a multitude of potential conditions of the threat stream may result, all of which must be evaluated.

There are five features needed to define the dynamic program. These are the definition of the state, a stage, the recursive relationship, the initialization conditions and the stopping conditions. Each of these is detailed below. Briefly, the state for this model is a set of four variables : the configuration of the threat stream, the problem time, the level of SEASPARROW resources remaining and the readiness of the RAM launcher. These variables define the system's capacity to respond at each stage of the problem. Each opportunity to assign the SEASPARROW system to a threat is a stage of the model. These and the remaining dynamic programming features employed in SCAM are detailed below.

SCAM obeys the following logic. At every possible system state, SCAM must evaluate the probability of survival resulting from every potential SEASPARROW assignment at that stage of the problem. The program searches through the threats present to calculate which targets are in range for a SEASPARROW assignment. SCAM calculates the time of flight of a SEASPARROW salvo directed at that target, hereafter referred to as the cycle. The program then determines which threats in the stream, if any, will be within RAM range during that time. These targets are classified Eligible. Any threat missile that has sufficient time to impact the defending ship during the cycle is classified as Critical.

SCAM calculates the probability of survival for all potential outcomes of the combined SEASPARROW decision and all the potential RAM and CIWS engagements that occur during the cycle. Theses calculations are repeated for SEASPARROW salvos of zero, one or two missiles. The SEASPARROW assignment which yields the best probability of surviving is stored and SCAM moves to the next threat missile that satisfies SEASPARROW engagement criteria.

A. STATE VARIABLES

Each SEASPARROW decision point represents a stage of the dynamic program. At each of these stages the decision made is based on a characterization of the state of the system, and the state is updated as a result of the decision. The SEASPARROW decision policy that maximizes the probability of survival at each stage is sought. The state of the system is (C, T, S, U), where:

C : The state of the threat stream. This state variable is a binary code which carries the relative positions of threats which are still present and those that have been neutralized. For example C=1101 indicates that the second missile has been destroyed. Likewise, C=1000 indicates that only the fourth target remains. This state variable has 2^n possible values, where n is the number of threat missiles in the raid.

T : The time since problem start. This state variable tracks the problem time and is used to update the positions of the threat missiles. State variables C and T combined with a list of the targets' original distances from the defending ship provide the necessary characterization of the threat raid. The nominal threat scenario is about 100 seconds long.

S : The remaining SEASPARROW missile inventory. The short duration of the problem precludes emptying the RAM magazine and so the RAM inventory does not need to be tracked. Additionally, the crude model of CIWS assumes unlimited magazine capacity for that system. SEASPARROW assignment decisions are expected to change as resources become scarce, and so the remaining inventory must be tracked. SEASPARROW salvos are limited to a maximum of two or the remaining inventory, whichever is least. The nominal SEASPARROW loadout is eight missiles.

U : The time until the RAM launcher is ready to fire. RAMs firing delay is the pivotal constraint on the system. As stated, without this delay and given the assumptions on engageability, RAM would have little difficulty handling the threat alone. This variable tracks the time penalty the system must pay if the RAM launcher is not ready at the moment a SEASPARROW assignment is made. In other words, due to some previous SEASPARROW assignment, the RAM launcher has fired a shot within the launcher delay

interval from the time of SEASPARROW intercept. As a consequence, once the new SEASPARROW decision has been made, RAM must wait the remaining launcher delay, U, before engaging any Critical or Eligible threats.

Both the time since problem start variable T, and the RAM delay variable U are modeled with increments of one second. For a scenario of four threat missiles, a full eight round SEASPARROW magazine, a maximum five second RAM delay and total problem duration of 100 seconds, SCAM must calculate the outcome of $2^4 \times 8 \times 5 \times 100 = 72$ thousand decisions. Early trials with SCAM used increments as small as 0.1 second, however the results were not significantly different and the model paid a significant penalty in terms of run time.

B. MODEL PARAMETERS

The following are the parameters used in the dynamic program:

V_{LF}	Velocity of a supersonic sea skimming cruise missile in miles/second,
V_{LS}	Velocity of a subsonic sea skimming cruise missile in miles/second,
V_{SS}	Velocity of the SEASPARROW missile in miles/second,
V_{RAM}	Velocity of the Rolling Airframe Missile (RAM) in miles/second,
RAM_{Min}	Minimum intercept range for RAM in miles,
SS_{Min}	Minimum intercept range for SEASPARROW in miles,
RAM_{Int}	Maximum intercept range for RAM in miles,
SS_{Int}	Maximum intercept range for SEASPARROW in miles,
DistM(i)	Distance in miles to the i^{th} threat missile at problem start DistM(1)=Max_{SS} below,
$Dist(i)_t$	Distance in miles to the i^{th} threat missile at time t since problem start,
M	Number of threat missiles,
S_{MAG}	Number of SEASPARROW in the magazine at problem start,

T_{END} Time in seconds at end of problem,

V_{Tgt} Velocity of threat missile, equal to V_{LF} or V_{LS} depending on scenario,

Min_{RAM} Minimum RAM engagement range in miles at launch,

$$Min_{RAM} = RAM_{Min}\left(\frac{(V_{RAM}+V_{Tgt})}{V_{RAM}}\right) \tag{4.1}$$

Min_{SS} Minimum SEASPARROW engagement range in miles at launch,

$$Min_{SS} = SS_{Min}\left(\frac{(V_{SS}+V_{Tgt})}{V_{SS}}\right) \tag{4.2}$$

Max_{RAM} Maximum RAM engagement range in miles at launch,

$$Max_{RAM} = RAM_{Int}\left(\frac{(V_{RAM}+V_{Tgt})}{V_{RAM}}\right) \tag{4.3}$$

Max_{SS} Maximum SEASPARROW engagement range in miles at launch,

$$Max_{SS} = SS_{Int}\left(\frac{(V_{SS}+V_{Tgt})}{V_{SS}}\right) \tag{4.4}$$

TBLR Time between RAM launches in seconds,

TBLS Time between SEASPARROW launches in seconds,

P_{RAM} Single shot Probability of Kill for RAM,

P_{SS} Single shot Probability of Kill for SEASPARROW,

P_{CIWS} Single salvo Probability of Kill for CIWS.

Equations 4.1 through 4.4 reflect the fact that while intercept ranges are constant functions of the weapon system's performance, engagement ranges are functions of both defending and threat weapon velocities.

C. CRITICAL TARGET TABLE

Recall that for any SEASPARROW assignment the threat missiles may be classified into three categories: SEASPARROW targets, RAM Eligible targets and

RAM/CIWS Critical targets. Figure 4.3 illustrates a SEASPARROW assignment which results in classification of two Critical targets, shown in the box. Based on the time between SEASPARROW salvos, defined as τ, directed against the third threat missile, the first two threats have sufficient time to impact the defending ship. Calculation of τ is given by Equation 4.5, where i is the threat missile SEASPARROW is assigned against. These lead threats must be defeated by the RAM and/or CIWS systems or the ship will be hit.

$$\tau = TBLS + \left(\frac{Dist(i)_t}{V_{SS}+V_{Tgt}}\right) \tag{4.5}$$

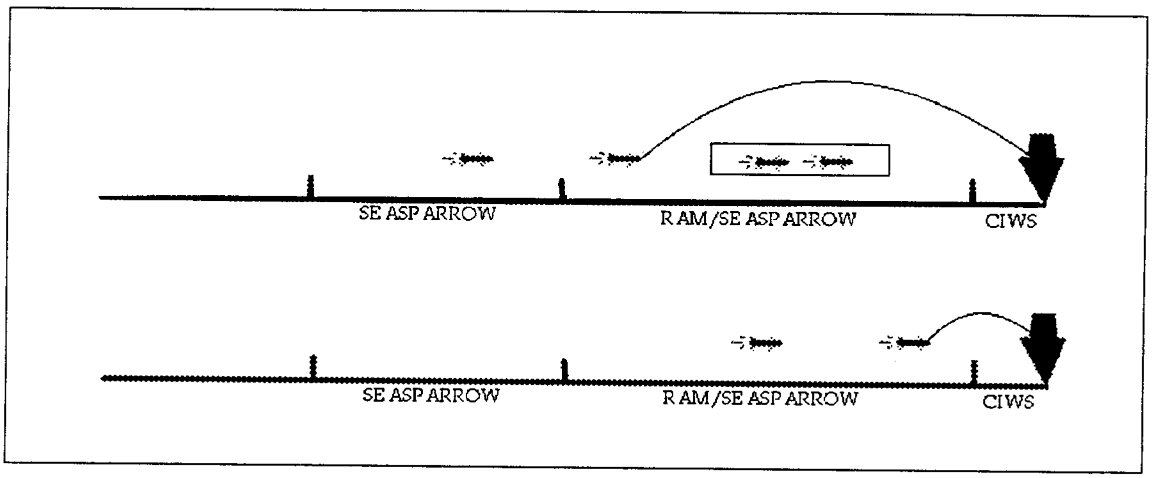

Figure 4.3

The distances to all threats at the time of the SEASPARROW assignment to target three are known. Consequently, the times until targets one and two impact the ship are known as well. Given the time between RAM salvos, TBLR, the time until the i^{th} threat hits the ship, t_i, and any residual launch delay resulting from a previous RAM shot, U, the number of RAMs N_i that can be fired against the i^{th} Critical target can be calculated. Equation 4.6 demonstrates this calculation.

$$N_i = \left\lceil \frac{(t_i - U)}{TBLR} \right\rceil \tag{4.6}$$

where $\lceil x \rceil$ is x rounded up to the nearest integer.

Equation 4.6 requires some additional clarification. Each N_i reflects the number of RAMs that may be fired at target i in t_i. $N_1 = 1$ and $N_2 = 2$ does not imply that the system may fire one RAM at the nearest Critical target and then two at the second Critical target. This condition does indicate that the system has time to fire one RAM before the nearest Critical target hits the ship, and a second RAM before the second Critical target impacts. The total number of RAMs that the ship may fire during a given SEASPARROW intercept is calculated using Equation 4.7.

$$N = \left\lceil \frac{(\tau - U)}{TBLR} \right\rceil \tag{4.7}$$

For all Critical targets i, N_i will be less than or equal to N. This is because in order for a threat to hit the ship, the SEASPARROW intercept time must be greater than the time needed for the Critical target to traverse the remaining distance.

Let $f_K(x;\ N_1,\ N_2,\ldots,\ N_K)$ be the probability that the ship survives all K Critical targets using exactly x RAM shots, hereafter referred to as the event E. If x is smaller than N, the total number of RAMs in τ, then N-x RAM shots will be available for Eligible targets. SCAM computes the function f_K and stores it for later use. The method of calculation is to condition on the value of the random variable R, the number of RAM shots that would be required to destroy the first Critical target if there were no constraint on the number used. If $R \leq x < N_1$, then E will happen if and only if the ship uses exactly x-R RAM shots in surviving all Critical targets after the first. This corresponds to RAM destroying the first Critical target. If $R \leq x$ and $x \geq N_1$ but $R \geq N_1$ then an additional way for E to occur is if the first N_1 RAM shots all miss, but the ship survives anyway and also uses $x - N_1$ RAM shots in surviving all Critical targets after the first. Assuming that all RAM shots are independent, $P(R=j) = P_{RAM}(1-P_{RAM})^{j-1}$ for $j \geq 1$, a Geometric distribution. Therefore, $f_K(x;\ N_1,\ N_2,\ldots,\ N_K)$ can be computed using the iterative formula given by Equation 4.8, with $f_0(x)=1$ if x=0, or $f_0(x)=0$ otherwise.

$$f_K(x;N_1,N_2,\ldots,N_K)=\begin{cases}\sum_{j=1}^{N_1}(P_{RAM}(1-P_{RAM})^{j-1}f_{K-1}(x-j;N_2-j,N_3-j)) & \text{if } x<N_1\\ \sum_{j=1}^{x}(P_{RAM}(1-P_{RAM})^{j-1}f_{K-1}(x-j;N_2-j,N_3-j)) & \\ +(1-P_{RAM})^{N_1}P_{CIWS}f_{K-1}(x-N_1;N_2-N_1,N_3-N_1) & \text{if } x\geq N_1\end{cases} \tag{4.8}$$

A simple example illustrates these calculations. For the situation illustrated in Figure 4.3, let the number of RAMs in τ be three. Let N_1 be zero, and N_2 be one. That is, there is not enough time to fire the RAM system at the nearest target, but enough time to fire once at the second. The probability of surviving with one RAM shot is $f_2(1;\,0,\,1)$, given by Equation 4.9. In this case, CIWS must handle the lead target successfully with probability $P_{CIWS} = 0.5$. The system will then have the opportunity to destroy the second target with a single RAM shot, with $P_{RAM} = 0.8$, or failing that a second CIWS salvo.

$$\begin{aligned} f_2(1;0,1) &= (0.2)^0(0.5)(0.9) = 0.45\\ f_1(1;1) &= P_{RAM}(1-P_{RAM})^0 f_0(0) + (1-P_{RAM})^1 P_{CIWS} f_0(0)\\ &= 0.8(0.2)^0(1.0) + (0.2)^1(0.5)(1.0) = 0.9\\ f_2(1;0,1) &= (1-P_{RAM})^0 P_{CIWS} f_1(1;1) \end{aligned} \tag{4.9}$$

The value of the stream state variable C, as a result of the decision to target the third threat missile with SEASPARROW is either 1000 or 1100, depending on the success of the SEASPARROW salvo. The lead two threats are destroyed irregardless of the outcome of the SEASPARROW intercept with the probability calculated above. Thus the probability the system has transitioned to a stream state of 1000 is the product of the Critical target probability calculated above, and the probability that the SEASPARROW salvo destroyed threat three. The probability the system is instead at stream state 1100 is the product of the Critical target probability with the probability that the SEASPARROW salvo missed threat three.

All the potential Critical target scenarios that can result from any SEASPARROW assignment can be enumerated. The number of potential Critical targets is bounded by the number of threats in the scenario. The number of RAMs that could potentially be fired at any threat missile is bounded as well by the limits of the problem duration and the

launcher cycle time of the RAM system. Given these bounds the f_K table can be precalculated and used as a lookup table for any SEASPARROW assignment decision in the model.

These calculations also highlight the disadvantage the combined defense suffers as a result of the launch delay for the RAM system. The most stressing condition the system will face is the near simultaneous arrival of multiple threats in the Critical category. Two threats for which the RAM has time to launch only a single missile obviously create more difficulty than threats which are spaced sufficiently to allow RAM an engagement opportunity against each. Minimizing tight group arrivals will consequently boost survival probabilities.

Critical targets are only one category of threat that can be handled this way. Engagement of threats which are classified as Eligible to the RAM system based on the SEASPARROW assignment can be determined similarly.

D. ELIGIBLE TARGET TABLES

An Eligible target is defined as one which is within RAM engagement range during the flight of the SEASPARROW salvo. Like Critical targets threats are determined to be Eligible based on the SEASPARROW assignment. Figure 4.4 illustrates a SEASPARROW assignment that results in classification of two Eligible threat missiles. Unlike the calculations for Critical targets, the system may have partial or no success against a set of threats which are Eligible. Thus the array of potential values for the stream state variable C as a result of a SEASPARROW assignment is considerably more complicated with Eligible targets than for Critical targets alone. Because all Critical targets must be defeated, the possibilities for the value of C are reduced to two: the value for a SEASPARROW success and all Critical targets destroyed, and the value for a SEASPARROW failure and all Critical targets destroyed.

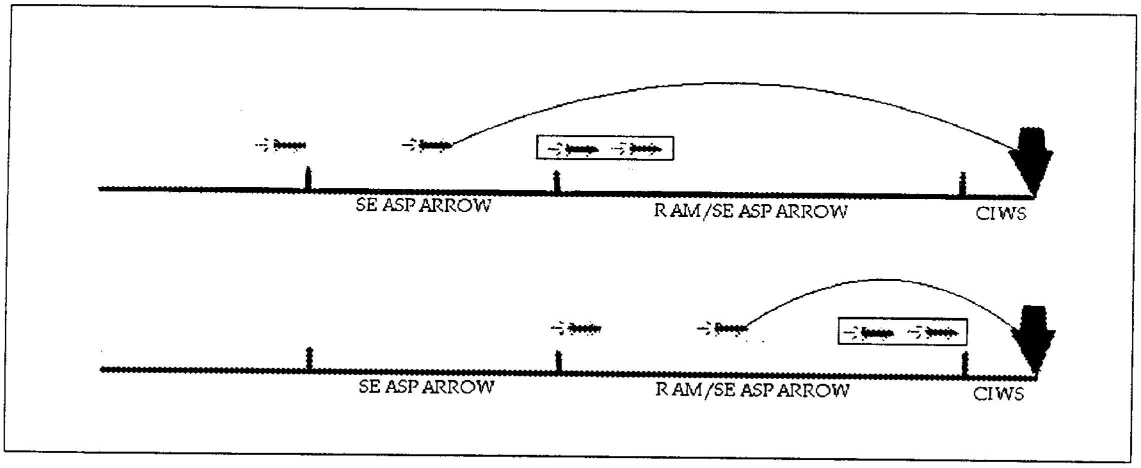

Figure 4.4

Eligible targets introduce a multitude of other values for the stream state. Using the scenario presented in Figure 4.4, the potential values for the stream state as a result of a SEASPARROW assignment against the third threat are :

- 1111, all engagements failed
- 1110, SEASPARROW failed but RAM destroyed the lead target
- 1100, SEASPARROW failed but RAM destroyed the first two targets
- 1011, SEASPARROW succeeded, but RAM failed
- 1010, SEASPARROW succeeded, RAM destroyed the lead target
- 1000, SEASPARROW and RAM both successfully destroyed all threats engaged

The diagram of potential outcomes when the SEASPARROW assignment results in both Critical and Eligible threat determinations is more complicated still. Referring to these potential results it is clear that RAM could destroy both Eligible threats with a single missile each. The RAM launcher delay, and more specifically the current state of the residual delay before RAM may fire, state variable U, is critical to all calculations of RAM effectiveness. The residual delay value impacts the number of RAMs that may be fired in τ. A wait of two seconds before the launcher may fire could be the difference between a single RAM shot at a Critical target, or no RAM shot. Threat missiles are classified as Eligible if they are within the RAM engagement envelope at the start of a SEASPARROW cycle. Thus SCAM does not consider a case where RAM is active for some part of the

cycle, pauses while waiting for subsequent targets to enter RAM engagement range and then resumes firing. Consequently, RAM is engaged continuously throughout the first part of each cycle, or possibly through all of it. In the latter case the next value of U depends only on the length of the cycle, and in the former the next value of U is zero. This results in a further division of the calculations for RAM success against Eligible targets.

What is needed are the probabilities for survival in the current stage with RAM firing throughout the time cycle, and with RAM defeating the Critical and Eligible targets without using all available shots. Let $g_L^0(y,j)$ be the probability that the ship kills the first j out of L Eligible targets when y RAM shots are available, without using the last RAM shot, hereafter referred to as event E^0. Let $g_L^1(y,j)$ be the probability of the same event except that the last RAM shot is used, hereafter referred to as event E^1. The method of calculation is to condition on the value of Z, the number of RAM shots in y that would kill their target, assuming no constraint on the number of targets. If $y \le L$, there is no possibility of event E^0 occurring and, event E^1 will occur if and only if Z=j. Assuming that all RAM shots are independent, $P(Z=j) = \binom{y}{j} P_{RAM}^j (1-P_{RAM})^{y-j}$, a Binomial distribution. Thus for $y \le L$, $g_L^0(y,j)$ and $g_L^1(y,j)$ can be computed using the formula given by Equation 4.10.

$$P(E^0) = g_L^0(y,j) = 0 \qquad\qquad 0 \le j \le y \le L$$

$$P(E^1) = g_L^1(y,j) = \binom{y}{j} P_{RAM}^j (1-P_{RAM})^{y-j} \qquad\qquad 0 \le j \le y \le L \tag{4.10}$$

If $y > L$ and $j \le L-1$ then again there is no possibility of event E^0 occurring, and event E^1 will occur if and only if Z=j. Thus for $y > L$ and $j \le L-1$, $g_L^0(y,j)$ and $g_L^1(y,j)$ can be computed using the formula given by Equation 4.11.

$$P(E^0) = g_L^0(y,j) = 0 \qquad\qquad 0 \le j < L < y$$

$$P(E^1) = g_L^1(y,j) = \binom{y}{j} P_{RAM}^j (1 - P_{RAM})^{y-j} \qquad\qquad 0 \le j < L < y \tag{4.11}$$

If y > L and Z = L = j then there are two possible outcomes. If the y^{th} RAM shot is a good shot and kills the j^{th} target then event E^1 occurs. If the y^{th} RAM shot is a bad RAM shot, since Z = L = j, all Eligible targets are destroyed before the last RAM shot and event E^0 occurs. But $P(E^0 | Z = L = j) = \frac{y-L}{y}$ and $P(E^1 | Z = L = j) = \frac{L}{y}$. Thus for y > L and Z = L = j, $g_L^1(y,L)$ can be computed by conditioning on the event Z = L using the formula given by Equation 4.12.

$$P(E^1) = P(E^1 | Z = L) P(Z = L)$$

$$g_L^1(y,L) = \left(\tfrac{L}{y}\right)\binom{y}{L} P_{RAM}^L (1 - P_{RAM})^{y-L} = \binom{y-1}{L-1} P_{RAM}^L (1 - P_{RAM})^{y-L}; 0 \le j = L < y \tag{4.12}$$

If Z > L there are more than sufficient good RAM shots in y to defeat all L Eligible targets, and so $P(E^0 | Z > L) = 1$. Assuming all RAM shots are independent, $P(Z>L) = \sum_{j=L+1}^{y} \binom{y}{j} P_{RAM}^j (1 - P_{RAM})^{y-j}$. Thus, $g_L^0(y,L)$ is given by Equation 4.13.

$$P(E^0) = P(E^0 | Z = L) P(Z = L) + P(E^0 | Z > L) P(Z > L)$$

$$\begin{aligned} g_L^0(y,L) &= \left(\tfrac{y-L}{y}\right)\binom{y}{L} P_{RAM}^L (1 - P_{RAM})^{y-L} + \sum_{j=L+1}^{y} \binom{y}{j} P_{RAM}^j (1 - P_{RAM})^{y-j} \\ &= \binom{y-1}{L} P_{RAM}^L (1 - P_{RAM})^{y-L} + \sum_{j=L+1}^{y} \binom{y}{j} P_{RAM}^j (1 - P_{RAM})^{y-j}; 0 \le j = L < y \end{aligned} \tag{4.13}$$

These calculations for events E^0 and E^1 can be predetermined for all possible Eligible target configurations in the same way the Critical target table was precalculated. The number of Eligible targets at any stage is bounded by the number of threats in the stream. The number of RAM shots available is bounded by the limits of the longest SEASPARROW intercept time, i. e., a maximum range intercept, and the fratricide delay

for the RAM launcher. Thus all possible engagements can be precalculated and stored in tables which then are referenced in the SEASPARROW recursion formulation.

An example illustrates the calculations:

Let $P_{RAM} = 0.8$, $L = 2$, $y = 4$

$$j = 0 \Rightarrow \qquad g_L^0(4,0) = 0$$
$$g_L^1(4,0) = \binom{4}{0}(.8)^0(.2)^4 = 0.001$$

$$j = 1 \Rightarrow \qquad g_L^0(4,1) = 0$$
$$g_L^1(4,1) = \binom{4}{1}(.8)^1(.2)^3 = 0.025$$

$$j = 2 = L \Rightarrow \qquad g_L^0(4,2) = \binom{3}{2}(.8)^2(.2)^2 + \binom{4}{3}(.8)^3(.2)^1 + \binom{4}{4}(.8)^4(.2)^0 = 0.$$
$$g_L^1(4,2) = \binom{3}{1}(.8)^2(.2)^2 = 0.076$$

E. SEASPARROW RECURSION FORMULATION

Any particular SEASPARROW assignment may result in classification of both Critical and Eligible targets. The resulting states from the SEASPARROW assignment encompass all the combinations of potential states resulting from a SEASPARROW success or failure, removal of all Critical targets and all the possible outcomes of RAM engagement of Eligible targets. The outcome of the SEASPARROW assignment is independent of the RAM engagements and may be handled separately. Since all K Critical targets must be defeated, the only difficulty is the calculation of the number of Eligible targets that have been destroyed.

Let $Q^0(j)$ be the probability of surviving all K Critical targets and also killing the first j out of L Eligible targets, and not using the last RAM shot, implying there is no residual RAM launcher delay. Let $Q^1(j)$ be the probability of surviving all K Critical targets and also killing the first j out of L Eligible targets and the last RAM shot is used, implying there is some residual RAM launcher delay, U. Refer to these two events as E^0 and E^1, and recall that N is the total number of RAM shots available during the current SEASPARROW salvo time of flight. Assuming that all RAM shots and CIWS salvos are independent, these joint probabilities can be computed by multiplying the individual event

probabilities. Event E^0 will occur if all Critical targets are destroyed with zero RAM shots and the first j Eligible targets are destroyed with the remaining N RAM shots. Or, event E^0 will occur if all Critical targets are destroyed with one RAM shot and the first j Eligible targets are destroyed with the remaining N-1 RAM shots, etc. Each of these event probabilities are mutually exclusive and so additive. It is clear that the probabilities for event E^1 are computed the same way. Thus, $Q^0(j)$ and $Q^1(j)$ can be calculated using the formulas given by Equations 4.14 and 4.15

$$Q^0(j) = \sum_{x=0}^{N-j} f_K(x; N_1, N_2, ..., N_K) g_L^0(N-x, j) \quad j = 0, 1, ..., L \tag{4.14}$$

$$Q^1(j) = \sum_{x=0}^{N-j} f_K(x; N_1, N_2, ..., N_K) g_L^1(N-x, j) \quad j = 0, 1, ..., L \tag{4.15}$$

At each stage SCAM examines the SEASPARROW assignment options against each threat in the stream within SEASPARROW range. This assignment in turn determines the classification of all threats in the stream as the SEASPARROW target, RAM Eligible targets and Critical targets. The option to not assign SEASPARROW to any target at that stage is also examined. What follows is an enumeration of the potential states resulting from each possible SEASPARROW assignment and the formulas to calculate the probability of survival for each.

Let h[C,S,T,U] be the maximum possible probability of survival given the state of the threat stream is C, there are S SEASPARROW missiles remaining, the problem has progressed T seconds from the start and the RAM launcher must wait U seconds before it can fire the next shot. Examining the case where SEASPARROW chooses not to engage any threat, there are three possible scenarios: RAM is not ready to fire due to some residual launcher delay, RAM is ready to fire but no threat satisfies launch criteria, RAM is ready to fire and some threat satisfies launch criteria. In each case the problem is advanced one second, the minimum time step, and a new stage is evaluated. If RAM fires and hits, the threat stream state is updated by removing the lead target. Figure 4.5

illustrates these conditions and the outcomes of each. C' is the new stream state with the lead threat removed.

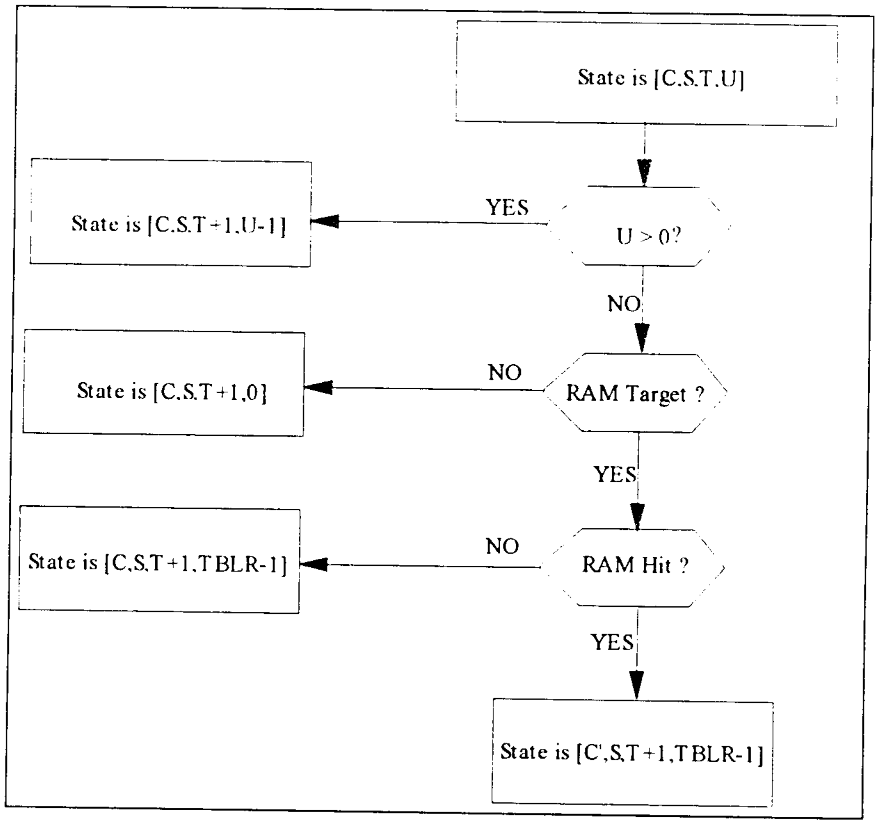

Figure 4.5

If a SEASPARROW assignment is made, let:

- C' be the stream state with the SEASPARROW target removed.
- C'_{K+j} be the stream state with the SEASPARROW target, all K Critical targets and j Eligible targets removed
- C^{+}_{K+j} be the stream state with all K Critical targets and j Eligible targets removed, meaning the SEASPARROW salvo failed to destroy the assigned target
- K be the number of Critical targets as a result of the SEASPARROW assignment
- L be the number of Eligible targets as a result of the SEASPARROW assignment
- τ be the time of the SEASPARROW engagement cycle. N be the number of RAM salvos in τ
- U' be the residual delay if the last RAM shot is needed to defeat the j^{th} threat as calculated by Equation 4.16

$$U' = MAX\{0, \lfloor U + (N+1)TBLR - \tau \rfloor\} \tag{4.16}$$

The ship survives if SEASPARROW hits its assigned target and RAM hits all K Critical targets and the first j out of L Eligible targets and if it survives the subsequent state, or if SEASPARROW misses its assigned target and RAM hits all K Critical targets and the first j out of L Eligible targets and if it survives the subsequent state. The probabilities $Q^0(j)$ and $Q^1(j)$ are mutually exclusive and lead to different states of the residual RAM launcher delay, U. Therefore, h[C,S,T,U] for a SEASPARROW salvo of one round can be computed iteratively using Equation 4.17, with h[0,S,T,U]=1.0, $\forall$ S, T, U.

$$\begin{aligned} h[C,S,T,U] = MAX\{\sum_{j=0}^{L}[(P_{SS})(Q^0(j))h[C'_{K+j}, S-1, T+\tau, 0] \\ +(1-P_{SS})(Q^0(j))h[C^{+}_{K+j}, S-1, T+\tau, 0] \\ +(P_{SS})(Q^1(j))h[C'_{K+j}, S-1, T+\tau, U'] \\ +(1-P_{SS})(Q^1(j))h[C^{+}_{K+j}, S-1, T+\tau, U']]\} \end{aligned} \tag{4.17}$$

The formula for the two round SEASPARROW salvo is similar with the P_{SS} term replaced by the probability of success in two shots, $1-(1-P_{SS})^2$, and the $(1-P_{SS})$ term replaced by the probability of failure in two shots, $(1-P_{SS})^2$. The maximum in

Equation 4.17 is taken over all feasible salvo sizes at each of the surviving threats in C. h[C,S,T,U] is then stored for use in subsequent calculations.

F. IMPLEMENTATION

SCAM was coded in MODSIM II. This was driven by the need for a programming language capable of handling the large array sizes needed by this application. SCAM was written using an Object Oriented Programming approach, though this is more a feature of the language than through any hope of reusing code modules.

Nominal scenario run times for SCAM are in the neighborhood of 15 minutes for a four threat scenario. Run times grow significantly with increases in the number of threats.

Initialization of SCAM consists of four phases. Phase one is the scenario input phase. Scenario input includes the number of threats in the raid (M), the type of threat missile, either subsonic or supersonic, the arrival window duration (W), initial magazine loadout for SEASPARROW (S_{MAG}), Probabilities of Kill for each weapon, and weapon delays for RAM (TBLR) and SEASPARROW (TBLS).

Phase two initializes the distances to the threat missiles. Initial threat missile distances are calculated using the user input for arrival window length in seconds (W), the number of threats present (M) and a random number seed. The closest threat missile is assigned the maximum SEASPARROW engagement distance Max_{SS}. Uniform random numbers are drawn from the interval [0,W]. These values are multiplied by threat missile velocity (V_{Tgt}) to calculate the separation distance. These distances are sorted in ascending order and assigned to threat missiles two through M-1. The last missile is assigned an initial distance of $Max_{SS} + (V_{Tgt})(W)$. These distances are stored in an array, DistM(i). Distance to the i^{th} threat at any time, t during the problem is calculated using Equation 4.18.

$$Dist(i)_t = DistM(i) - (V_{Tgt})(t) \tag{4.18}$$

It should be noted that unlike a Monte Carlo simulation, SCAM obtains the same results for a given set of scenario parameters regardless of how many times it is run. Random numbers are used only in setting up the scenario.

The third initialization phase is the precalculation phase for Critical and Eligible targets. Because the maximum number of threats that can be classified as Critical or Eligible is bounded by the number of threats present, and the maximum number of RAM shots available in any cycle is bounded as well, $f_k(x; N_1, N_2, \ldots, N_K)$, $g_L{}^0(y,j)$ and $g_L{}^1(y,j)$ can all be precalculated and stored. These calculations take a few seconds to perform.

The final initialization step is to initialize the value of h[0,S,T,U] = 1.0 for all possible values of S, T and U. This allows the recursion formula for SEASPARROW to start.

V. RESULTS

SCAM determines the optimal assignment of the SEASPARROW system at each stage. Both HUGHES [Ref 6] and Powell [Ref 7] assigned SEASPARROW independent from RAM, and since the maximum engagement range of SEASPARROW is longer, the result was to fire a SEASPARROW salvo at the lead threat missile for a maximum range intercept. In contrast, SCAM frequently chooses to wait for some later target to enter the SEASPARROW engagement envelope before firing its first salvo.

Recall that the baseline scenario is four Exocet missiles in a 20 second window. This scenario is consistent with the conditions analyzed by HUGHES and Powell, and serves as a convenient starting point to test SCAM's assignment policy. The baseline set of input parameters includes this description of the raid density and the probabilities of kill for the component weapon systems, and firing delays for SEASPARROW and RAM. These were chosen to reflect relative performance between weapon systems, yet remain unclassified. The base set of parameters is as follows:

- SEASPARROW PK is 0.6
- RAM PK is 0.8
- CIWS PK is 0.5
- SEASPARROW firing delay is ten seconds
- RAM firing delay is five seconds

These values are not the true performance figures for these systems. They do preserve the relative performance of these weapons. Given the assumptions on engageability made in SCAM, the higher speed and maneuverability of the Rolling Airframe Missile grants a higher level of effectiveness [Ref 8]. Firing delays reflect the system features developed in Chapter II.

A. THREAT SPACING

Recall that by selecting a different random number seed the pattern of interarrival distances within the threat stream is manipulated. Given the constraints on the RAM system imposed by the launcher delay, the interarrival distance of threats within the stream is expected to have a dramatic effect on the SEASPARROW assignment decisions. In

order to minimize the potential for overwhelming the RAM/CIWS system for Critical targets, SEASPARROW should look for groups of targets in the missile stream to attack. In effect, SEASPARROW should build gaps in the missile stream to relieve the burden simultaneous arrivals places on RAM and CIWS.

For the base case of four Exocets in 20 seconds, several different patterns of threat spacing were tested, including equal interarrival distances. When interarrival distances are the same, SCAM assigns the initial SEASPARROW salvo to the lead target. However, when interarrival distances are dissimilar, SCAM's policy changes. Figure 5.1 illustrates a sample of four different raid configurations at the first SEASPARROW assignment opportunity, when the lead target is at Max_{SS}. The arrow indicates the first threat SCAM assigned to the SEASPARROW system.

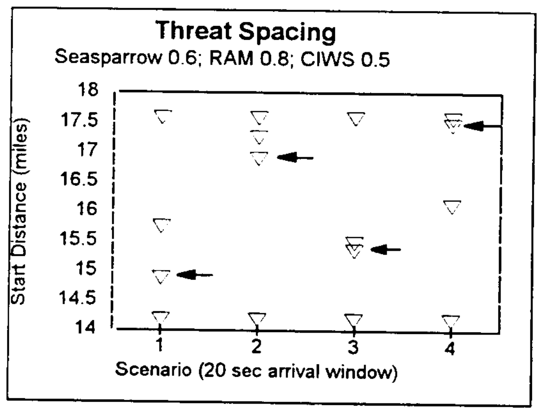

Figure 5.1

In each of these sample cases, the optimal SEASPARROW assignment was to wait and not simply engage the lead threat, but rather to find the two closest threats in the stream and engage the nearest of the two. By targeting the tightest pair, the SEASPARROW builds gaps in the stream.

This policy of shooting holes in groups that allows RAM and CIWS to defend against the remaining threat more easily is robust across a wide range of scenario conditions. For a small raid size of only two missiles, obviously the policy is identical to

the uncoordinated assignment policy and SCAM targets the lead threat. For stressing raid densities, the policy holds as well. Figure 5.2 shows the results of two different cases with six Exocet missiles arriving in twenty seconds. This scenario is more difficult for RAM, and consequently the assignment of SEASPARROW to mutually support RAM is more crucial than in the nominal threat scenarios.

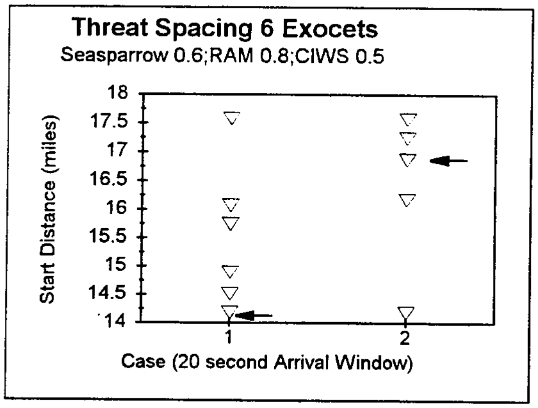

Figure 5.2

In Case 1, the first two missiles were the closest together and so the lead target was engaged by SEASPARROW. In Case 2, the third and fourth targets were nearly simultaneous, and so plotted as a single target, and SCAM held fire with SEASPARROW until target three was in range. These sample outcomes reveal the optimal assignment policy for the SEASPARROW: build gaps in the threat stream to assist RAM and CIWS. This policy is accomplished by identifying the closest pair of threat missiles and assigning SEASPARROW against the lead missile in the pair. The question of obvious interest is whether this policy holds true for a range on scenario conditions, and if not what features is the policy sensitive to.

B. ARRIVAL WINDOW

The SEASPARROW assignment policy is expected to be sensitive to the duration of the arrival window from the first threat in the stream to the last. Holding all other scenario inputs constant, compressing the arrival window has the affect of stressing the

RAM system more by decreasing the interarrival distances between all threats. Expanding the arrival window increases the interarrival distance between all threats. To evaluate SCAM's policy for sensitivity to changes in the duration of the arrival window, two alternate windows are tested: a total arrival window of ten seconds, and thirty seconds. These conditions are consistent with the HUGHES simulation model. The relative positions of the threats in the stream are preserved, their actual interarrival distances change as the arrival window changes.

The results are presented in the same format as for threat spacing. Figure 5.3 illustrates the initial SEASPARROW assignment for each of three threat patterns in each of the three arrival windows. In each scenario, the policy of engaging the tightest grouped pair in the stream held for the ten second arrival window. However, in the thirty second arrival window cases, SCAM assigned SEASPARROW to the lead target. The mean probability of survival for four Exocets arriving over a thirty second arrival window was 0.99997 compared to a mean probability of survival in the same condition without SEASPARROW of 0.99984. Four Exocets in thirty seconds is not a stressing case for the RAM system, and so the assignment of SEASPARROW makes little difference.

Changes in the arrival window make no difference for raids of only two missiles, but for raids of six Exocets the results are illuminating. Figure 5.4 shows SCAM's SEASPARROW assignment policy for six missile raids over the three arrival windows. Note that unlike the four missile case, the six missile case is not trivial for RAM at the thirty second window. SEASPARROW is assigned to the lead missile in the closest pair for all three arrival windows.

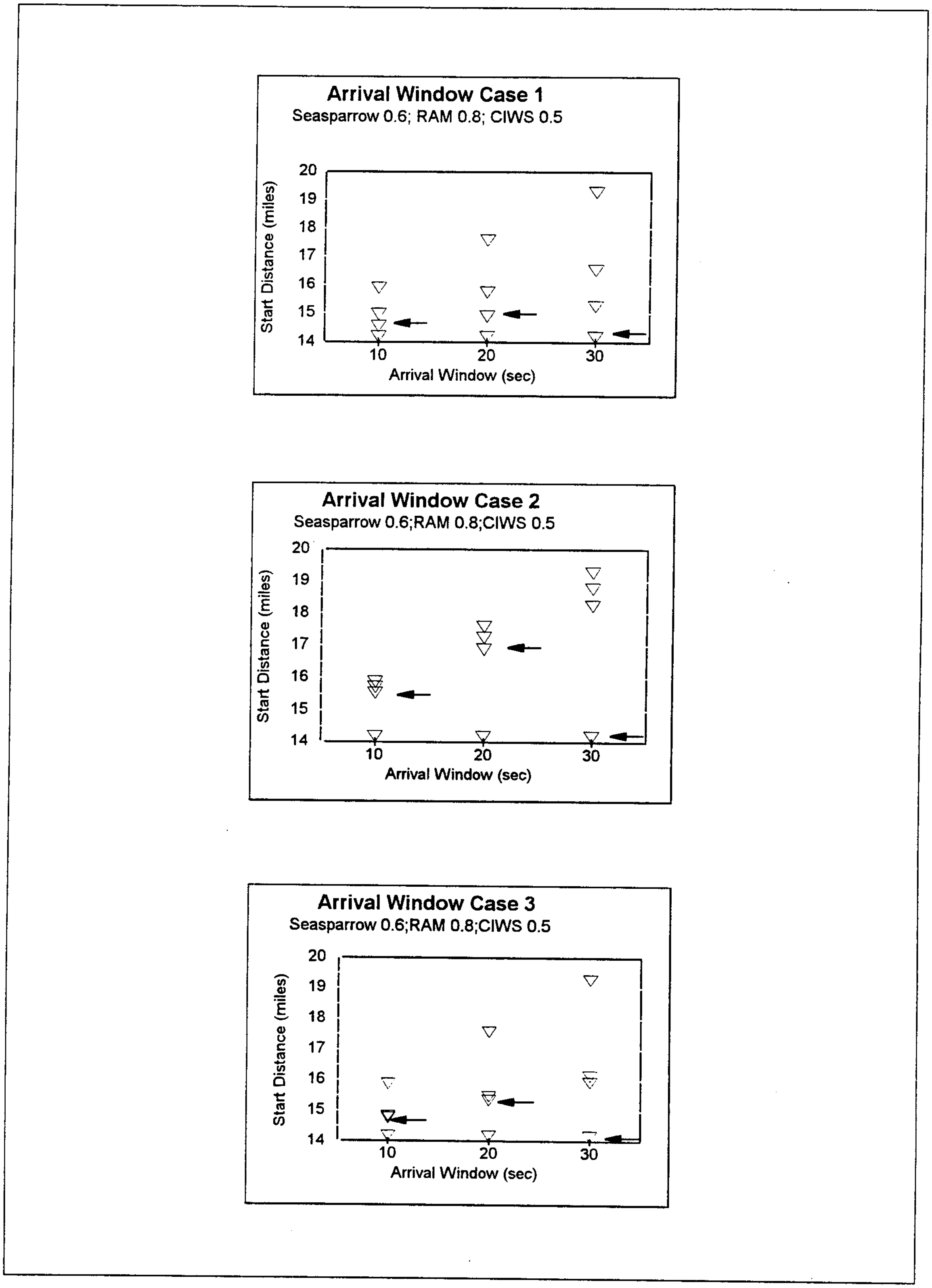
Arrival Window Case 1
Seasparrow 0.6; RAM 0.8; CIWS 0.5
Start Distance (miles)
20
19
18
17
16
15
14
10
20
30
Arrival Window (sec)
Arrival Window Case 2
Seasparrow 0.6;RAM 0.8;CIWS 0.5
Start Distance (miles)
20
19
18
17
16
15
14
10
20
30
Arrival Window (sec)
Arrival Window Case 3
Seasparrow 0.6;RAM 0.8;CIWS 0.5
Start Distance (miles)
20
19
18
17
16
15
14
10
20
30
Arrival Window (sec)

Figure 5.3

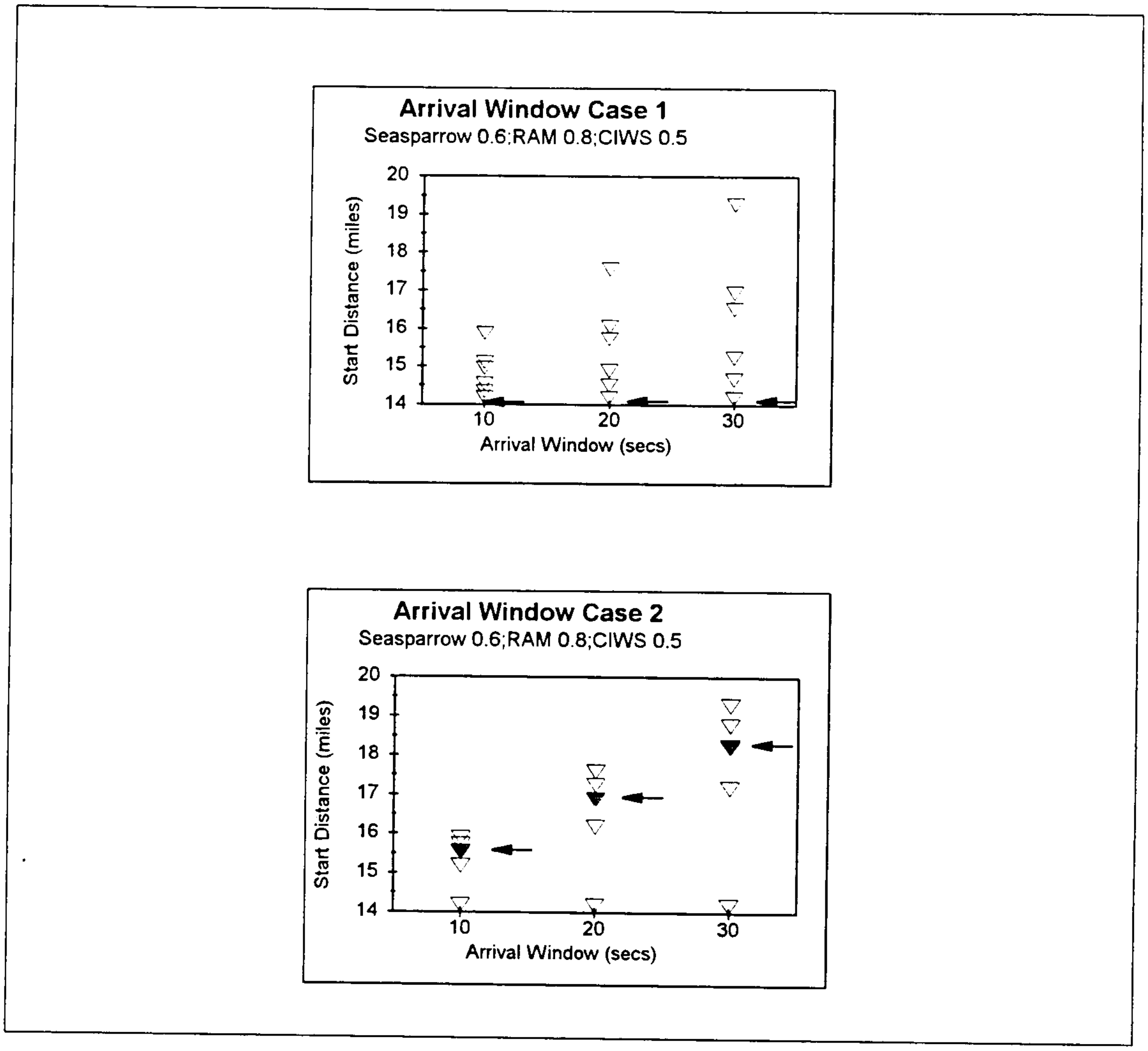

Figure 5.4

C. SEASPARROW PROBABILITY OF KILL

The assignment policy for SEASPARROW has thus far been sensitive to the spacing of threat missiles in the target stream, and to the duration of the arrival window. Under the base case parameters, SEASPARROW has a lower probability of kill than RAM: 0.6 compared to 0.8. Holding all other parameters constant, the SEASPARROW policy is tested for sensitivity to changes in the effectiveness of the SEASPARROW missile. In all cases tested the SEASPARROW assignment policy was always to fire a two missile salvo at the target missile, yielding a probability of hitting the target of $1-(1-P_{SS})^2$.

Figure 5.5 demonstrates the results when the probability of kill for SEASPARROW was varied to 0.4 and 0.8. From these results it is clear that the optimal policy determined by SCAM is robust for a wide range of probabilities of kill for SEASPARROW. The assignment for SEASPARROW in the six Exocet case proved equally insensitive to changes in the effectiveness of the SEASPARROW system.

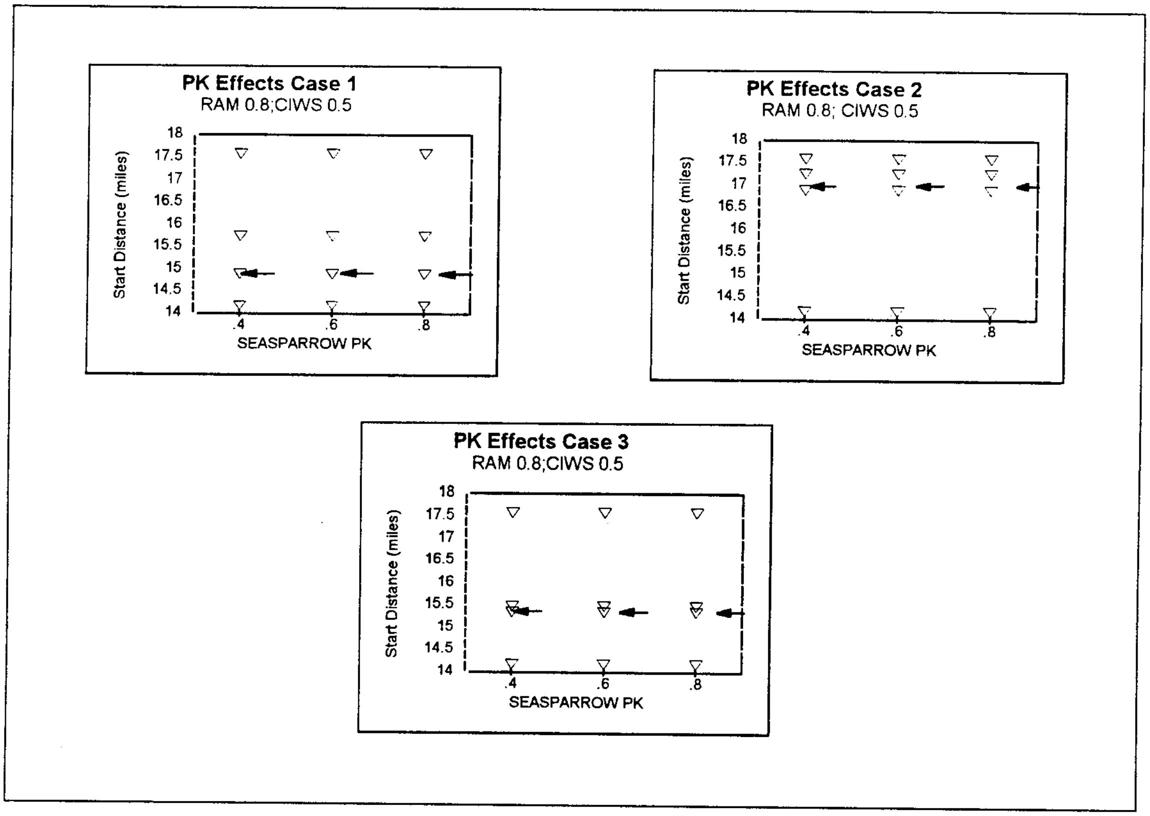

Figure 5.5

D. RAM DELAY

Finally the SEASPARROW policy determined by SCAM was tested against changes in the duration of the RAM launcher delay. The previous results indicate that the RAM launcher delay is the limiting factor on RAMs capability and drives the SEASPARROW assignment toward building gaps in the stream. The base value for the RAM launcher delay is five seconds. Figure 5.6 provides the optimal assignment of the

initial SEASPARROW salvo with all other conditions held constant and the RAM launcher delay varied from two to six seconds.

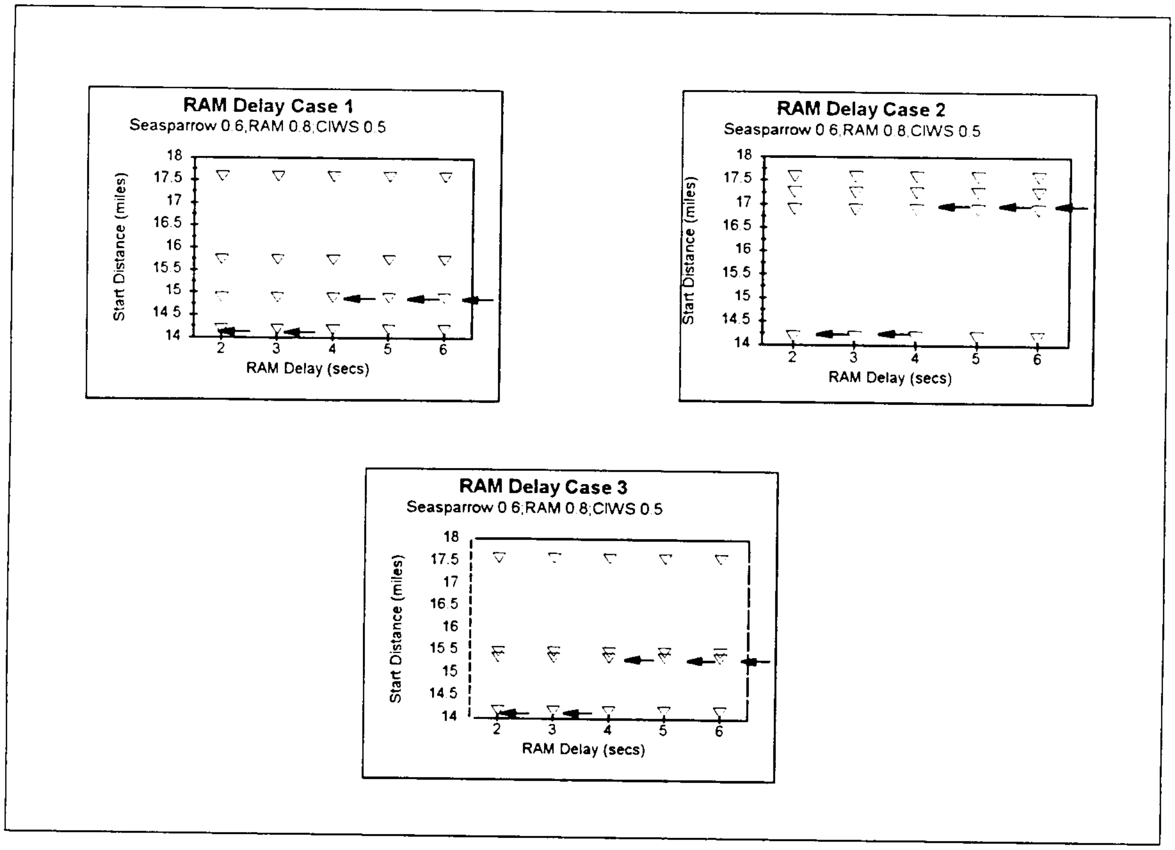

Figure 5.6

These sample cases indicate that SCAM's assignment policy is sensitive to the duration of the RAM launcher delay. As RAM launcher delay becomes short and crosses some threshold, SEASPARROW returns to engaging the lead threat.

E. HIGH SPEED THREAT

It was pointed out in Chapter I that the proliferation of supersonic anti-ship cruise missiles is not nearly of the same magnitude as for subsonic threats. Yet, supersonic anti-ship missiles do exist and currently pose a significant threat to air defense systems. SCAM includes the supersonic sea skimming cruise missile profile. The policy of targeting groups within the stream is equally appropriate for the supersonic threat, as the number of engagement opportunities for both SEASPARROW and RAM declines.

Indeed, since RAMs capability against the high speed threat is more limited due to fewer engagement opportunities, the assignment decision for SEASPARROW is even more important. Figure 5.7 shows an example of the assignment decision for the initial SEASPARROW salvo against a raid of four supersonic sea skimming cruise missiles.

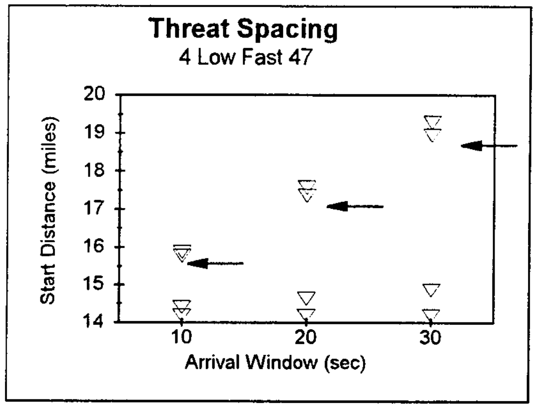

Figure 5.7

Note that for the high speed threat, the assignment policy is unchanged for the ten, twenty or thirty second arrival windows.

VI. CONCLUSION

SCAM routinely found that the optimal policy for SEASPARROW assignment was to not shoot at the lead target, but instead to shoot holes in groups of threat missiles. By exposing SCAM to a variety of threat scenarios and conducting sensitivity analysis on the input conditions, this policy was found to be robust for a large set of threat scenarios. In fact, SEASPARROW was fired at the lead target only for cases where the RAM launcher delay was three seconds or less, or when the pattern of threat missiles allowed for an intercept of the lead threat before a group entered engagement range, or when the lead target was part of a tightly spaced group. These results build a strong argument that the coordination policy best suited to this system, and the one that provides the best mutually supportive use of the SEASPARROW missile system is to target the SEASPARROW at groups of threat missiles in the stream.

SCAM highlights two ways to improve the probability of survival of an anti-ship cruise missile attack against the DD-963 with the NATO SEASPARROW, the Rolling Airframe Missile and the Phalanx Close-In Weapon System. First, coordinate the assignment of the SEASPARROW to mutually support the RAM and CIWS installation by shooting at groups of threat missiles. Second, since RAM is the main line of defense, reduce the RAM fratricide delay, so as to increase the rate of fire and limit the need for SEASPARROW to build gaps in the target stream.

A. MODEL EXCURSIONS

SCAM is limited to handling attacks on a single bearing. Modification of the dynamic program to handle attacks from multiple bearings would be a relatively simple addition, that could potentially provide insight into a broader range of threat scenarios. This might indicate that SEASPARROW should target threat missiles so the remaining targets were near the same line of bearing so RAM would not need to train its launcher to engage multiple targets. In effect rather than building gaps in the target pattern, concentrating the remaining threat for RAM and CIWS.

The HUGHES simulation used range dependent probabilities of kill. SCAM could easily be modified to include this feature. Another possible area where the assignment policy would be expected to change is in the case of heterogeneous threat configurations or mixtures of subsonic and supersonic threat missiles. This would have the affect of changing the interarrival distance between targets as the problem progressed from start to finish.

B. RECOMMENDATIONS

While it is not universally the optimal policy employed by SCAM, shooting the SEASPARROW at the lead target in the tightest spaced pair of threat missiles neatly summarizes the SEASPARROW assignment decision most frequently made. This has the added advantage that an engagement rule of this nature could easily be employed.

The long run time of SCAM, approximately fifteen minutes for the nominal scenario, precludes using this tool for anything more than as a means to gain insight into the potential benefit of coordinating these weapons for mutual support. The next step is to develop or modify a simulation to employ the coordination policy SCAM reveals and compare empirical results from this simulation to those obtained from the uncoordinated firing policy. Trusting that results from such an effort will reinforce the results obtained by SCAM, the policy of firing SEASPARROW at the lead target of the tightest pair of detected threats seems a simple coordination rule that could easily be implemented on warships.

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